The inevitable nonlinearity of quantum gravity falsifies the many-worlds interpretation of quantum mechanics

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Abstract

There are fundamental reasons as to why there should exist a reformulation of quantum mechanics which does not refer to a classical spacetime manifold. It follows as a consequence that quantum mechanics as we know it is a limiting case of a more general nonlinear quantum theory, with the nonlinearity becoming significant at the Planck mass/energy scale. This nonlinearity is responsible for a dynamically induced collapse of the wave-function, during a quantum measurement, and it hence falsifies the many-worlds interpretation of quantum mechanics. We illustrate this conclusion using a mathematical model based on a generalized Doebner-Goldin equation. The non-Hermitian part of the Hamiltonian in this norm-preserving, nonlinear, Schrödinger equation dominates during a quantum measurement, and leads to a breakdown of linear superposition.

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There are two fundamental unsolved problems in our understanding of quantum mechanics. The first is the famous problem of quantum measurement, for which one of the possible solutions is the mechanism of decoherence, in conjunction with the many-worlds interpretation of quantum mechanics. An alternative explanation of a quantum measurement is a dynamically induced collapse of the wave-function, which requires modification of the Schrödinger equation in the measurement domain. The second unsolved fundamental problem is the need for a reformulation of quantum mechanics, which does not refer to a classical spacetime manifold [1]. In this essay we show that these two unsolved problems have a deep connection, and the resolution of the second problem implies that quantum measurement is explained by dynamically induced collapse of the wave-function. This, in turn, falsifies the many-worlds interpretation of quantum mechanics.

The standard formulation of quantum theory depends on an external classical time. The need for a reformulation of quantum mechanics which does not refer to a classical spacetime manifold arises because the geometry (metric and curvature) of the manifold is produced by *classical* matter fields. One can envisage a Universe in which there are only quantum, and no classical, fields. This will cause the spacetime geometry to undergo quantum fluctuations, which, in accordance with the Einstein hole argument, destroy the underlying classical spacetime manifold. However, one should still be able to describe quantum dynamics; hence the need for the aforementioned reformulation. The new formulation becomes equivalent to standard quantum mechanics as and when an external classical spacetime geometry becomes available.

When one tries to construct such a reformulation of quantum mechanics, it follows from very general arguments [1] that quantum gravity is effectively a nonlinear theory. What this means is that the 'quantum gravitational field' acts as a source for itself. Such a nonlinearity cannot arise in the standard canonical quantization of general relativity, which is inherently based on linear quantum theory, and which leads to the Wheeler-DeWitt equation. It also follows as a consequence that at the Planck mass/energy scale, quantum theory itself becomes an effectively nonlinear theory [because of self-gravity], and that the Hamiltonian describing a quantum system depends nonlinearly on the quantum state. The standard linear quantum theory is recovered as an approximation at energy scales much smaller than the Planck mass/energy scale.

In [1] we have developed a model for the above-mentioned reformulation of quantum mechanics, based on noncommutative differential geometry. One of the outcomes of this model is that the non-relativistic

quantum mechanics of a particle of mass m is described by a nonlinear Schrödinger equation, which belongs to the Doebner-Goldin class [2] of nonlinear equations. The nonlinear terms depend on the mass of the particle, and are extremely small when the particle's mass is much smaller than Planck mass $m_{Pl} \sim 10^{-5}$ grams. Thus in the microscopic domain the theory reduces to standard quantum mechanics. The nonlinearity becomes significant in the mesoscopic domain, where the particle's mass is comparable to Planck mass. This is also the domain where the quantum to classical transition is expected to take place; a nonlinearity in this domain can play a decisive role in explaining quantum measurement. It is pertinent to mention here that current experimental tests of quantum mechanics do not rule out such a nonlinearity, and furthermore, because our model is based on an underlying noncommutative geometry, the usual objections against a nonlinear quantum mechanics do not apply [1]. When the particle's mass is greater than Planck mass, the nonlinear theory reduces to standard classical mechanics.

We now demonstrate how the Doebner-Goldin equation can explain quantum measurement as dynamical collapse of the wave-function. The simplest D-G equation is

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi + iD(m/m_{Pl})\hbar\left(\nabla^2\psi + \frac{|\nabla\psi|^2}{|\psi|^2}\psi\right). \tag{1}$$

The coefficient D of the nonlinear, imaginary, part of the Hamiltonian is a real constant, which depends on the ratio of the particle's mass to Planck mass. D goes to zero in the limit $m \ll m_{Pl}$, so that then the D-G equation reduces to the linear Schrödinger equation. As m approaches m_{Pl}, D becomes large enough for the imaginary part of the Hamiltonian to dominate over the real part. The equation is norm-preserving, although the probability density obeys not the continuity equation, but a Fokker-Planck equation. The equation is of interest also because it arises in the study of unitary representations of an infinite-dimensional Lie algebra of vector fields $Vect(R^3)$ and group of diffeomorphisms $Diff(R^3)$ - these representations provide a way to classify physically distinct quantum systems. Further, the equation is a special case [3] of the following class of norm-preserving nonlinear Schrödinger equations

$$i\hbar d|\psi>/dt = H|\psi> + (1-P_\psi)U|\psi> \tag{2}$$

where H is the Hermitian part of the Hamiltonian, $(1-P_\psi)U$ is the non-Hermitian part, $P_\psi = |\psi><\psi|$ is the projection operator, and U is an arbitrary nonlinear operator. We will work with a generalization of the U operator for this D-G equation, given by $U = iF(m/m_{Pl})\Sigma_n D_n U_n$, where

$$U_n = \left[\frac{<\psi|\nabla|\chi_n><\chi_n|\nabla|\psi>}{<\psi|\chi_n><\chi_n|\psi>}|\chi_n><\chi_n| + \nabla^2\right] \tag{3}$$

and where D_n are state-dependent scalars; the real function $F(m/m_{Pl})$ vanishes as $m \rightarrow 0$ and monotonically increases with mass, and $|\chi_n>$ are a complete set of orthonormal vectors.

We will use the term 'initial system' to refer to the quantum system $\mathcal{Q}$ on which a measurement is to be made by a classical apparatus $\mathcal{A}$, and the term 'final system' to refer jointly to $\mathcal{Q}$ and $\mathcal{A}$ after the initial system has interacted with $\mathcal{A}$. A quantum measurement will be thought of as an increase in the mass (equivalently, number of degrees of freedom) of the system, from the initial value $m_{\mathcal{Q}} \ll m_{Pl}$ to the final value $m_{\mathcal{Q}} + m_{\mathcal{A}} \gg m_{Pl}$. Clearly then, the non-Hermitian part in (2), which is proportional to U, and hence to the scalars D_n in (3), will play a critical role in the transition from the initial system to the final system.

We assume that $\mathcal{A}$ measures an observable $\hat{O}$ of $\mathcal{Q}$, having a complete set of eigenstates $|\phi_n>$. Let the quantum state of the initial system be given as $|\psi>= \Sigma_n\, a_n|\phi_n>$. The onset of measurement corresponds to mapping the state $|\psi>$ to the state $|\psi>_F$ of the final system as

$$|\psi>\rightarrow |\psi>_F \equiv \sum_n a_n|\psi>_{Fn}= \sum_n \; a_n|\phi_n>|A_n> \tag{4}$$

where $|A_n>$ is the state the measuring apparatus would be in, had the initial system been in the state $|\phi_n>$, and the $|\chi_n>$ in (3) should be understood as the direct product $|\chi_n>=|\phi_n>|A_n>$.

During a quantum measurement the non-Hermitian part of the Hamiltonian in (2) dominates over the Hermitian part, and governs the evolution of the state $|\psi>_F$ given by (4). Assuming that the Hermitian operator U_n maps the state $|\psi>_F$ to a state $|\xi>_{nF}$ which can be expanded as

$$|\xi>_{nF}= \sum_m \; b_{nm}|\phi_m>|A_m> \tag{5}$$

we substitute the expansion for $|\psi>_F$ from (4) in (2), and neglecting the Hermitian part of the Hamiltonian we get [3]

$$\frac{da_n}{dt} = \frac{F(m/m_{Pl})}{\hbar}\; a_n(q_n - L) \tag{6}$$

where $q_n = t_n/a_n$, $L = \Sigma_m\, a_m^* t_m$, $t_m = \Sigma_s b_{ms} D_s$. If the dependence of the D_n's on the state is such that the q_n's are *random constants* then it follows that [3]

$$\frac{d}{dt}\left(\ln \frac{a_i}{a_j}\right) = \frac{F(m/m_{Pl})}{\hbar}\; [q_i - q_j]. \tag{7}$$

It follows that only the state $|\psi>_{Fi}$ having the largest real part of q_i survives at the end of a measurement (since $\Sigma_n|a_n|^2 = 1$), and in this manner superposition is broken. It is noteworthy that the time-scale for breakdown of superposition is directly proportional to Planck's constant, and it decreases with increasing mass.

The randomness of the q_n's is needed to ensure that repeated measurements of the observable $\hat{O}$ lead to different outcomes $|\psi>_{Fn}$. In order to reproduce the observed Born probability rule, the measurement should cause the quantum system to collapse to the eigenstate $|\phi_n>$ with the probability $p_n = |<\psi(t_0)|\phi_n>|^2$. The most plausible way to introduce randomness in the q_n's is to propose that they are related to the random phase θ_0 of the initial quantum state. As an example, if the phase is uniformly distributed in the range $[0, 2\pi]$ and the q_n's are related to θ_0 by the relations [3]

$$q_1 = -2\pi\theta_0, \quad q_n = -\frac{1}{n}\left(2\pi\theta_0 - \sum_k^{n-1} |<\psi(t_0)|\phi_k>|^2\right) - \sum_k^{n-1} \frac{|<\psi(t_0)|\phi_k>|^2}{k} \tag{8}$$

and possess the probability distribution

$$\omega(q_n) = |<\psi(t_0)|\phi_n>|^2 \ \exp(|<\psi(t_0)|\phi_n>|^2) \tag{9}$$

the Born probability rule is reproduced.

The detailed assumptions of the above model can only be justified after a better understanding of the relation between quantum mechanics and noncommutative geometry has been achieved. However, it is already clear that the natural requirement of a reformulation of quantum mechanics which does not refer to a classical spacetime manifold compels us to consider a nonlinear modification of the Schrödinger equation at the Planck mass/energy scale. Such a nonlinearity, which explicitly depends on Newton's gravitational constant (via the Planck mass) is responsible for the breakdown of superposition during a quantum measurement, and provides a dynamical explanation for collapse of the wave-function. Modifications of the Schrödinger equation hitherto investigated in the literature have been *ad hoc*, and introduced solely for the purpose of explaining quantum measurement. However, the nonlinear modification considered by us has its origin elsewhere, in quantum gravity; yet it has an impact on quantum measurement.

The experimentally observed mechanism of decoherence destroys the *interference* between different possible outcomes of measurement, but as it is based on standard linear quantum mechanics, it preserves *superposition* amongst the alternatives. In this scheme (assuming that the wave-function describes indi-

vidual quantum systems, and not merely their statistical ensemble), the only natural way to explain the observed lack of superposition amongst the results of a measurement is to invoke the many-worlds interpretation of quantum mechanics, wherein upon a measurement, the Universe splits into many branches, one for every decohered state. Up until now, no theoretical argument had been presented, to choose between a decoherence based explanation of quantum measurement, and the alternative explanation based on dynamically induced collapse. Our analysis in this essay establishes that the wave-function does collapse during a measurement, and hence the many-worlds interpretation stands falsified. Above all, the proposal that the initial random phase of the quantum state is correlated with the outcome of a quantum measurement is experimentally testable with current generation experiments, and if confirmed, will provide the first experimental evidence for quantum gravity.

References

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